• constraints

• competing optimization requirements

MPC provides a systematic, unified solution to problems with these characteristics.

1.3.1 SOME EXAMPLES

Example I: Blending systems (input constraints)

- control $r_A$ & $r_B$ (first priority).
- control $q$ if possible (second priority).
- possibility of valve saturation must be taken into account.
Classical Solution:

MPC Solution:

At $t=k$, solve

$$
\min_{u_i} \sum_{i=1}^{p} \left\| \begin{bmatrix} (r_A)_{k+i|k} \\ (r_B)_{k+i|k} \end{bmatrix} - \begin{bmatrix} (r_A)_{ref} \\ (r_B)_{ref} \end{bmatrix} \right\|_Q^2 + \left\| q_{k+i|k} - q_{ref} \right\|_R^2
$$

$$Q \gg R$$

$$
\begin{bmatrix}
(u_1)_{min} \\
(u_2)_{min} \\
(u_3)_{min}
\end{bmatrix} \leq \begin{bmatrix}
(u_1)_j \\
(u_2)_j \\
(u_3)_j
\end{bmatrix} \leq \begin{bmatrix}
(u_1)_{max} \\
(u_2)_{max} \\
(u_3)_{max}
\end{bmatrix}, \quad j = 0, \ldots, p - 1
$$
Example II: Two-point control in a distillation column (input constraints, interaction)

- strong interaction
- “wind-up” during saturation
- saturation of an input requires recoordination of the other input
**Clasical Solution:** Two single-loop controllers with anti-windup scheme (decouplers not shown)

- $T_1$ controller does not know that $V$ has saturated and vice versa $\Rightarrow$ coordination of the other input during the saturation of one input is impossible.
- mode-switching logic is difficult to design / debug (can you do it?) and causes ”bumps”, etc.
MPC Solution:

At $t = k$, solve

$$
\min_{\Delta \mathbf{u}_i} \sum_{i=1}^{p} \left\| \begin{bmatrix} (T_1)_{k+i|k} \\ (T_2)_{k+i|k} \end{bmatrix} - \begin{bmatrix} (T_1)_{\text{ref}} \\ (T_2)_{\text{ref}} \end{bmatrix} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \begin{bmatrix} \Delta L_{k+i|k} \\ \Delta V_{k+i|k} \end{bmatrix} \right\|_R^2
$$

with

$$
\begin{bmatrix} L_{\text{min}} \\ V_{\text{min}} \end{bmatrix} \leq \begin{bmatrix} L_{k+i|k} \\ V_{k+i|k} \end{bmatrix} \leq \begin{bmatrix} L_{\text{max}} \\ V_{\text{max}} \end{bmatrix}
$$

for $i = 0, \ldots, m - 1$

- easy to design / debug / reconfigure.
- anti-windup is automatic.
- optimal coordination of the inputs is automatic.
Performance of classical solution vs. MPC

SISO loops w/ anti-windup & decoupler (no mode switching):

MPC:
Example III: Override control in compressor (output constraint)

- control the flowrate
- but maintain $P \leq P_{\text{max}}$

Classical Solution:
MPC Solution:

At \( t = k \), solve

\[
\min_{\Delta u_i} \sum_{i=1}^{p} \left\| q_{k+i|k} - q_{\text{ref}} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2
\]

with

\[
P_{k+i|k} \leq P_{\text{max}} \quad \text{for } i = 1, \cdots, p
\]
Example IV: Override control in surge tank (output constraints)

- control the outlet flowrate
- but maintain $L \geq L_{\text{min}}$

Classical Solution:
MPC Solution:

At $t = k$, solve

$$\min_{\Delta u_i} \sum_{i=1}^{p} \left\| q_{k+i|k} - q_{ref} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2$$

with

$$L_{k+i|k} \geq L_{min} \quad \text{for} \quad i = 1, \ldots, p$$
Example V: Valve position control in air distribution network (optimization requirement)

- control the flowrates of individual channels
- minimize the air compression

Classical Solution:

![Diagram of an air distribution network with valve position control](image)
MPC Solution:

At $t = k$, solve

$$
\min_{\Delta u_k} \sum_{i=1}^p \left\| \begin{bmatrix}
(q_1)_{k+i|k} \\
\vdots \\
(q_n)_{k+i|k}
\end{bmatrix} - \begin{bmatrix}
(q_1)_{ref} \\
\vdots \\
(q_n)_{ref}
\end{bmatrix} \right\|^2_Q + \sum_{i=1}^{m-1} \| P_{k+i|k} - P_{min} \|^2_R
$$

with $Q \gg R$ and

$$
\begin{bmatrix}
P_{min} \\
(u_1)_{min} \\
\vdots \\
(u_n)_{min}
\end{bmatrix} \leq \begin{bmatrix}
P_{k+i|k} \\
(u_1)_{k+i|k} \\
\vdots \\
(u_n)_{k+i|k}
\end{bmatrix} \leq \begin{bmatrix}
P_{max} \\
(u_1)_{max} \\
\vdots \\
(u_n)_{max}
\end{bmatrix}
$$

for $i = 0, \cdots, m - 1$
Example VI: Heavy oil fractionator (all of the above)

- $y_7$ must be kept above $T_{\text{min}}$.
- $y_1$ and $y_2$ is to be kept at setpoint (measurements delayed).
- BRD must be minimized to maximize the heat recovery.
Classical Solution:

Not clear

- how to use temperature measurements to fight the effect of delays, unreliability, etc. of analyzers.
- how to accommodate the optimization requirement.

MPC Solution:

\[
\min \sum_{i=1}^{p} \left\| \begin{bmatrix} y_1 \\ y_2 \\ u_3 \end{bmatrix}_{k+l|k} - \begin{bmatrix} y_1 \\ y_2 \\ u_3 \end{bmatrix}_{ref} \right\|^2 + \sum_{i=1}^{m} \left\| \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{bmatrix}_{k+l|k} \right\|^2
\]

\[y_7 \geq T_{min}\]

plus other input constraints.
Example VII: Tennessee Eastman process (supervisory control requirements)

\[
\min \sum_{i=1}^{p} \left\| \begin{bmatrix} Q \\ G/H \end{bmatrix}_{k+i|k} - \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}_{k+i|k} Q \right\|_2^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2
\]

\[P_r \leq (P_r)_{\text{max}}\]
\[(H_r)_{\text{min}} \leq H_r \leq (H_r)_{\text{max}}\]

where
\[P_r: \text{reactor pressure}, \quad (P_r)_s: \text{setpoint to reactor pressure loop}\]
\[H_r: \text{reactor level}, \quad (H_r)_s: \text{setpoint to reactor level loop}\]
\[Q: \text{total product flow} \quad G/H: \text{mass ratio between products G and H}\]
\[F_D: \text{D feed flow} \quad F_E: \text{E feed flow}\]
1.3.2 SUMMARY

Advantages of MPC over Traditional APC

- control of processes with complex dynamics
- decoupling and feedforward control are “built in” (traditional approaches are difficult for systems larger than $2 \times 2$).
- constraint handling
- utilizing degrees of freedom
- consistent methodology
- realized benefits: higher on-line times and cheaper implementation / maintenance