Chapter 3

SYSTEM IDENTIFICATION

The quality of model-based control absolutely relies on the quality of the model.

3.1 DYNAMIC MATRIX IDENTIFICATION

3.1.1 STEP TESTING
Procedure

1. Assume operation at steady-state with

controlled var. (CV): \( y(t) = y_0 \) for \( t < t_0 \)

manipulated var. (MV): \( u(t) = u_0 \) for \( t < t_0 \)

2. Make a step change in \( u \) of a specified magnitude, \( \Delta u \) for

\[
u(t) = u_0 + \Delta u \quad \text{for} \quad t \geq t_0
\]

3. Measure \( y(t) \) at regular intervals:

\[
y_k = y(t_0 + kh) \quad \text{for} \quad k = 1, 2, \ldots, N
\]

where

\( h \) is the sampling interval

\( Nh \) is approximate time required to reach steady state.

4. Calculate the step response coefficients from the data

\[
s_k = \frac{y_k - y_0}{\Delta u} \quad \text{for} \quad k = 1, \ldots, N
\]

Discussions

1. Choice of sampling period

- For modeling, best \( h \) is one such that \( N = 30 \sim 40. \)

Ex: If \( g(s) = Ke^{-ds}/(\tau s + 1), \)
then settling time \( \approx 4\tau + d \)
Therefore, \( h \approx \frac{4\tau + d}{N} = \frac{4\tau + d}{40} = 0.1\tau + 0.025d \)
• May be adjusted depending upon control objectives.

2. Choice of step size (Δu)

• too small:
  
  May not produce enough output change
  Low signal to noise ratio

• too big:
  
  Shift the process to an undesirable condition
  Nonlinearity may be induced.
- Trial and error is needed to determine the optimum step size.

3. Choice of number of experiments
- Averaging results of multiple experiments reduces impact of disturbances on calculated $s_k$'s
- Multiple experiments can be used to check model accuracy by cross-validation.
  
  Data sets for Identification $\leftrightarrow$ Data set for Validation

4. An appropriate method to detect steady state is required.

5. While the steady state (low frequency) characteristics are accurately identified, high frequency dynamics may be inaccurately characterized.

3.1.2 PULSE TESTING

Procedure

1. Steady operation at $y_0$ and $u_0$.

2. Send a pulse of size $\delta u$ lasting for 1 sampling period.

3. Calculate pulse response coefficients

$$h_k = \frac{y_k - y_0}{\delta u} \quad \text{for} \quad k = 1, \ldots, N$$

4. Calculate the step response coefficients as a cumulative sum of $h_k$.

$$s_k = \sum_{i=1}^{k} h_i \quad \text{for} \quad k = 1, 2, \ldots, N$$
Discussions

1. Select $h$ and $N$ as for the step testing.
2. Usually need $\delta u \gg \Delta u$ for adequate S/N ratio.
3. Multiple experiments are recommended for the same reason as in the step testing.
4. An appropriate method to detect steady state is required.
5. Theoretically, pulse is a perfect (unbiased) excitation for linear systems.

3.1.3 RANDOM INPUT TESTING

Concept
Type of Inputs

1. Pseudo-Random Binary Signal (PRBS)

In MATLAB, \[ u = u_0 + \text{del} * 2 * \text{sign} \left( \text{rand} \left( 100, 1 \right) \right) - 0.5; \]
or \[ u = \text{mlbs} \left( 12 \right); \]

2. Random Noise

In MATLAB, \[ u = u_0 + \text{del} * 2 * \left( \text{rand} \left( 100, 1 \right) - 0.5 \right); \]
Data Analysis - Least Squares Fit

Given \( \{u_1, u_2, \ldots, u_M\} \) and \( \{y_1, y_2, \ldots, y_M\} \), determine the best fit FIR (finite impulse response) model \( \{h_1, h_2, \ldots, h_N\} \).

Consider

\[
y_k = h_1 u_{k-1} + h_2 u_{k-2} + \ldots + h_N u_{k-N} + d_k
\]

Assume the effects of initial condition are negligible.

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_M 
\end{bmatrix} =
\begin{bmatrix}
  u_0 & u_{-1} & \ldots & u_{1-N} \\
  u_1 & u_0 & \ldots & u_{2-N} \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{M-1} & u_{M-2} & \ldots & u_{M-N} 
\end{bmatrix}
\begin{bmatrix}
  h_1 \\
  h_2 \\
  \vdots \\
  h_N 
\end{bmatrix}
+ 
\begin{bmatrix}
  d_1 \\
  d_2 \\
  \vdots \\
  d_M 
\end{bmatrix}
\]

\[
y = Uh + d
\]

The least squares solution which minimizes

\[
(y - Uh)^T(y - Uh) = \sum_{i=1}^{M} \left( y_i - \sum_{j=1}^{N} h_j u_{i-j} \right)^2
\]

is

\[
\hat{h} = (U^T U)^{-1} U^T y
\]

In MATLAB, \( \gg \hat{hhat} = y \backslash U; \)

Discussions

1. Random input testing, if appropriately designed, gives better models than the step or pulse testing does since it can equally excite low to high frequency dynamics of the process.

2. If \( U^T U \) is singular, the inverse doesn’t exist and identification fails. → persistent excitation condition.
3. When the number of coefficients is large, $\mathbf{U}^T \mathbf{U}$ can be easily singular (or nearly singular). To avoid the numerical, a regularization term is added to the cost function. \( \rightarrow \) \textit{ridge regression}

$$\min_h [ (\mathbf{y} - \mathbf{U}h)^T (\mathbf{y} - \mathbf{U}h) + \alpha h^T h ]$$

$$\rightarrow \hat{h} = (\mathbf{U}^T \mathbf{U} + \alpha \mathbf{I})^{-1} \mathbf{U}^T \mathbf{y}$$

4. \textit{Unbiasedness}: If $d(\cdot)$ and/or $u(\cdot)$ is zero-mean and $u(i)$ is uncorrelated with $d(j)$ for all $(i, j)$ pairs (these conditions are easily satisfied.), the estimate is unbiased.

$$\hat{h} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T (\mathbf{U}h + d) = h + (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T d$$

Since

$$E\{(\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T d \} = 0$$

we have

$$E\{\hat{h}\} = h$$

5. \textit{Consistency}: In addition to the unbiasedness,

$$\hat{h} \rightarrow h \ (\text{or, equivalently } E\{(\hat{h} - h)(\hat{h} - h)^T\} \rightarrow 0) \text{ as } M \rightarrow \infty.$$

6. Extension to MIMO identification is straightforward.

The above properties are inherited to the MIMO case, too.
Example

- Process: $h = [h_1, h_2, h_3, h_4, h_5, \ldots] = [1.5, 2.0, 5.5, 0.1, 0, 0, \ldots]$

![FIR Coefficients of the Process](image)

- Input: PRBS with

\[ N = 200 \quad \bar{u} = 0 \]

![Input Sequence](image)
The resulting output response corrupt with measurement noise with 
\[ \sigma_n^2 = 0.25^2 \]
is

Estimates of \( \{h_j\} \) appear as
3.1.4 DATA PRETREATMENT

The data need to be processed before they are used in identification.

(a) Spike/Outlier Removal

- Check plots of data and remove obvious outliers (e.g., that are impossible with respect to surrounding data points). Fill in by interpolation.
- After modeling, plot of actual vs predicted output (using measured input and modeling equations) may suggest additional outliers. Remove and redo modelling, if necessary.
- But don’t remove data unless there is a clear justification.

(b) Bias Removal and Normalization

- The input/output data are biased by the nonzero steady state and also by disturbance effects. To remove the bias, difference is taken for the input/output data. Then the differenced data is conditioned by scaling before using in identification.

\[
\begin{align*}
y(k) &= \left( y^{\text{proc}}(k) - y^{\text{proc}}(k - 1) \right)/c_y \\
u(k) &= \left( u^{\text{proc}}(k) - u^{\text{proc}}(k - 1) \right)/c_u \\
\end{align*}
\] → identification
(c) Prefiltering

- If the data contain too much frequency components over an undesired range and/or if we want to obtain a model that fits well the data over a certain frequency range, data prefiltering (via digital filters) can be done.

The two filters should be same.