\[ A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})n(k) \]

\[
\begin{bmatrix}
A_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & A_n
\end{bmatrix}
\begin{bmatrix}
y_1(k) \\
\vdots \\
y_n(k)
\end{bmatrix}
= 
\begin{bmatrix}
B_{11} & \cdots & B_{1m} \\
\vdots & \ddots & \vdots \\
B_{n1} & \cdots & B_{nm}
\end{bmatrix}
\begin{bmatrix}
u_1(k) \\
\vdots \\
u_m(k)
\end{bmatrix}
+ 
\begin{bmatrix}
C_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & C_n
\end{bmatrix}
\begin{bmatrix}
n_1(k) \\
\vdots \\
n_n(k)
\end{bmatrix}
\]

\[ A_1(q^{-1})y_1(k) = B_{11}(q^{-1})u_1(k) + \cdots + B_{1m}(q^{-1})u_m(k) + C_n(q^{-1})n_1(k) \]

\[ A_n(q^{-1})y_n(k) = B_{n1}(q^{-1})u_1(k) + \cdots + B_{nm}(q^{-1})u_m(k) + C_n(q^{-1})n_n(k) \]

**FIR (Finite Impulse Response) Model**

\[ y(k) = H_1 u(k - 1) + \cdots + H_{n_b} u(k - n_b) + w(k) \]

where \( H_i \) is a impulse response coefficient (matrix) and \( w(k) \) is a zero-mean random noise (not necessarily i.i.d.).

- Cannot be used for description of unstable systems.
- Requires (much) more parameters than the corresponding ARMAX or state space model does.
  For description of a SISO stable system, usually 40 or more pulse response coefficients are needed if the sampling interval is appropriately chosen (not too short and too long).
- Irrespective of the nature of \( w(k) \) as far as it is of zero-mean, unbiased parameter estimates can be obtained using a simple least squares method.
State space model

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) + w(k) \\
y(k) &= Cx(k) + v(k)
\end{align*}
\]

where \( \{w(k)\} \) and \( \{v(k)\} \) are white noise sequences.

- Adequate to MIMO system description.
  Many useful canonical forms are well developed
- Powerful identification methods called the subspace method which
directly finds a state space model in a balanced form has been recently
developed.
  A version of the subspace method was commercialized by SETPOINT.

3.4 EXPERIMENTAL CONDITIONS

3.4.1 SAMPLING INTERVAL

- Too long sampling interval \( \rightarrow \) too much loss of information
  Too short sampling interval \( \rightarrow \) too much computation
- There are many different guidelines. \( h \approx \tau/10 \) is thought to be
  adequate for most applications.
3.4.2 OPEN-LOOP VS. CLOSED-LOOP EXPERIMENTS

Open-loop experiment

\[ u = \text{process input} \]
\[ y = \text{process output} \]

Closed-loop experiment

\[ u = \text{process input, controller output} \]
\[ y = \text{process output, controller input} \]
⇒ Identified Model ≈ Process or 1/Controller

- For nonparametric models (typically transfer functions),

\[ \hat{G}_{\text{model}}(s) \approx G_{\text{process}}(s) \text{ when } d = 0 \]
\[ \hat{G}_{\text{model}}(s) \approx G_{\text{control}}(s) \text{ when } v = 0 \]

- For parametric models (FIR, ARMAX, State Space ...),

\[ \hat{G}_{\text{model}}(s) \approx G_{\text{process}}(s) \]

if identifiability is satisfied.

Identifiability is in most case satisfied if

1. a FIR model is used and/or
2. a high-order controller is used and/or
3. independent excitation is given on \( v \).

### 3.4.3 INPUT DESIGN

- Remember that the excitation input has limited energy with finite magnitudes over a finite duration. Hence, it is inevitable that the identified model has biased information of the process.

- Depending on the way how to distribute the input energy over different frequencies and also over different input principal directions (for MIMO cases), the identified model may have different characteristics.

- The input should preferrably be designed to sufficiently excite the system modes which are associated with the desired closed-loop performance.

For a SISO process, process information near the crossover frequency is most important. The Ziegler-Nichols tuning method (i.e., continuous
cycling method) is justified in this sense.

- In general, the PRBS (Pseudo-Random Binary Sequence) is used as an excitation signal. By adjusting the minimum step length, we can change the frequency contents in the PRBS.